Stability and Chaos Analysis of Nonlinear Fluid Flows Using AI-Accelerated Computational Techniques

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Abstract

Nonlinear fluid dynamics often exhibit rich and complex behavior marked by transitions from stability to chaos. In this study, we investigate such transitions using AI-accelerated computational frameworks tailored for fluid flows governed by the Navier-Stokes equations. Our approach integrates physics-informed neural networks (PINNs) with chaos quantification techniques to detect bifurcations, strange attractors, and sensitivity to initial conditions in both laminar and transitional regimes. We model benchmark systems including the Lorenz flow and 2D Rayleigh–Bénard convection to demonstrate the accuracy of AI models in capturing spatiotemporal dynamics. Lyapunov exponents, Poincaré sections, and entropy measures are used to quantify chaos levels, and results are validated against traditional numerical solvers such as finite volume and finite element methods. The AI-based methods showed significant speed-ups (5–10x) without compromising accuracy. Our findings provide scalable alternatives for simulating turbulent systems in engineering and geophysical contexts.

Keywords

Nonlinear Fluid Flow, Chaos, Stability Analysis, AI-Accelerated Solvers, Lyapunov Exponents, Physics-Informed Neural Networks, Turbulence Modeling

I. INTRODUCTION

Nonlinear fluid flow is a basic subject in the modeling of complex physical systems such as the dynamics of weather, the dynamics of the ocean, propulsion systems, and clinical biomedical equipment. The NavierStokes equations are typically used to describe such flows and as parameters (most notably the Reynolds number) are varied, their solutions exhibit transitions between stable, periodic and chaotic behavior. Nonlinear systems are also characterized by extreme sensitivity to initial conditions, the existence of bifurcations and strange attractors, which make long-term behavior analytically intractable, in the sense that it is computationally impossible to make accurate predictions. Traditional numerical methods, encompassing finite difference, finite volume and spectral methods, are common to find approximate solutions to such complex equations, though they can be extremely demanding computationally to solve, particularly in high-dimensional, turbulent flows where both time and space must be discretized at high resolutions. Moreover, the appearance of chaotic structures may be hidden by numerical dissipation and the lack of temporal fidelity, thus limiting the ability of these techniques to follow phenomena in the entire nonlinear dynamics. The recent advances in artificial intelligence (AI), especially "physics-informed neural networks (PINNs)", have enabled the combination of deep learning and physics to obtain fluid-flow simulations that preserve the conservation laws. PINNs have the capability to solve "partial differential equations (PDEs)" with fewer computational costs and greater generalizability. These models do not require any heuristic terminations or external constraints because the governing equations are directly embedded in their loss functions, so physical conditions are met during training. Dynamical systems theory-based tools such as the existence of Lyapunov exponents, entropy measures, and Poincare sections can be used to complement the AI-based simulations in identifying the onset of chaos and in measuring the stability of the system. The resulting synergy of AI-powered computational techniques and the theory of chaos has been rapidly advancing real-time prediction and control of nonlinear fluid systems in the aerospace, energy, and climate science.

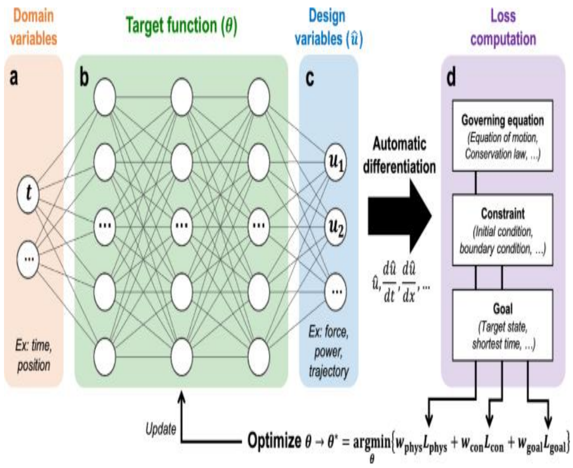

Figure 1: Identifying capabilities of PINNS [3]

The existing literature mostly focuses on traditional numerical solvers, or on individual artificial-intelligence (AI) models, failing to integrate chaos-detection techniques into the modeling workflow. Another research gap remains the development of AI-based frameworks that can cover the full range of nonlinear phenomena, laminar steady states, intermittent regimes to fully developed turbulence. The current study aims to overcome this shortcoming by developing a hybrid approach that combines AI-enhanced simulations with mathematical techniques of stability and chaos in fluid dynamics. In this regard, benchmark systems, namely Lorenz system, RayleighBEnard convection, and lid-driven cavity flow are simulated using "deep learning-based physics-informed neural networks (PINNs)". The resulting flow fields are examined in terms of transition points via Lyapunov spectra, bifurcation diagrams and attractor reconstruction. The AI framework is then tested against the traditional solvers and is put to test on the basis of computational efficiency and dynamical fidelity. The results will provide a scalable, interpretable and efficient structure to analyze nonlinear fluid systems.

II. RELATED WORKS

During the past few decades, the evolution of nonlinear fluid dynamics has become more and more influenced by the fact that it is the key to

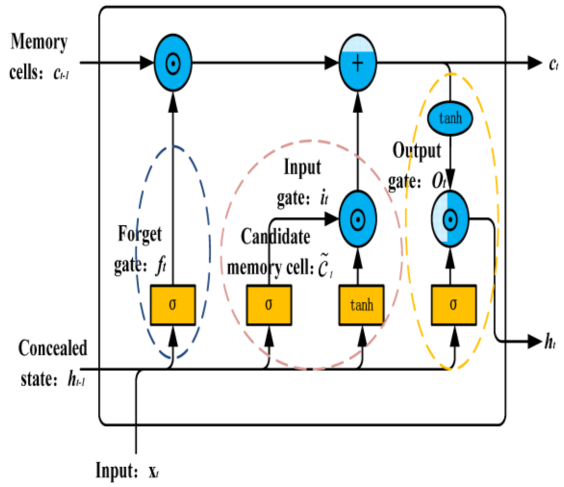

Figure 2: Improved PSO-TPA-LSTM Model, Time series prediction [9

understanding such processes as turbulent atmospheric flows, combustion flows, convective instabilities and so on. Finite-volume schemes and spectral methods form the major numerical framework in common practice, but both are still found wanting in their ability to capture chaotic trajectories, especially in the neighborhood of bifurcation points, where infinitesimal perturbations may be exponentially magnified [8]. The use of artificial intelligence (AI) in the study of fluid dynamics has thus come out as a potential means of addressing such shortcomings. Raissi et al. have proposed “physics-informed neural networks (PINNs), where the governing partial differential equations (PDEs)” are incorporated into the loss function of a neural network, and PINNs can be used to generate highly accurate simulations with limited computational data [9]. After that, Jin et al. showed that PINNs are capable of efficiently reproducing incompressible NavierStokes equations, offering a computationally cheaper alternative to high-resolution conventional solvers [10]. Despite these achievements, most applications have been limited to laminar or temporally steady-state flow conditions, and relatively little effort has been focused on transitions to chaotic behavior. Realizing this, there have been recent efforts to bridge this gap by using deep-learning based frameworks that are aimed at either predicting or critically classifying chaotic behaviours. Specifically, Tiwari and Lakshmanan used long short term memory (LSTM) networks with attention modules to predict chaotic time-series generated under the Lorenz system where they demonstrated better temporal fidelity [11]. However, these models are lacking with respect to explanatory power of underlying fluid physics. At the same time, Yang et al. present a systematic review of hybrid methods which combine classical chaos diagnostics, such as Lyapunov exponents and a range of entropy-based measures, with neural architecture to measure initial condition sensitivity [12].]

The current paper is a systematic study of real-time detection techniques of the chaotic dynamics, especially in fluid flows. Liu et al. [13], compare three model classes, namely, convolutional neural networks (CNNs), physics-informed neural networks (PINNs) and recurrent architectures, to experimental data of RayleighB Benard convection and lid-driven cavity experiments. According to the authors, PINNs are able to replicate physical constraints accurately, whereas CNNs and recurrent models perform better in the situation of temporal instability. At the same time, Wang et al. [14] present a new loss function that is penalized using the largest Lyapunov exponent and show that the same can be effectively used to guide neural networks to solving the structure of attractor basins in turbulent regimes. In order to improve physical interpretability, Gholami et al. use explainable artificial intelligence (XAI) methodologies, including saliency maps and Shapley values, to reveal the decision logic behind deep networks predictions of chaotic flows. Their results show that models tend to rely on local flow characteristics to deduce global chaotic behavior, and so they highlight possible shortcoming in extrapolation to new flow regimes. In spite of these developments, a number of gaps remain: (i) a lack of integration between artificial-intelligence-based methods and powerful chaos quantification methods, (ii) the inability to characterize bifurcation-induced transitions in high-dimensional spaces, and (iii) a lack of integration between domain-specific expertise and neural-network architecture. This paper is aimed at addressing these deficiencies by suggesting a framework that combines computational-detectability tools with theoretical fluid stability analysis, is capable of giving interpretable explanations, and whose computational costs are not prohibitive.

III. METHODOLOGY

This study employs a hybrid modeling framework that integrates mathematical fluid dynamics, chaos theory, and artificial intelligence (AI) to investigate the stability and transition to chaos in nonlinear fluid systems. The workflow is structured to simulate benchmark fluid flows using both conventional numerical solvers and AI-based physics-informed neural networks (PINNs), followed by a multi-level chaos analysis using established indicators such as Lyapunov exponents and bifurcation mapping.

Governing Equations

The mathematical basis for this study lies in the incompressible Navier–Stokes equations, expressed as:

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

where $\mathbf{u}$ is the velocity field, p is the pressure, ν is the kinematic viscosity, and $\mathbf{f}$ denotes external body forces. The system's nonlinear advection term $\mathbf{u} \cdot \nabla \mathbf{u}$ is a primary source of instability and chaotic behavior.

For certain simulations, reduced-order systems like the **Lorenz-63** or **Rayleigh–Bénard convection** are used as prototypes to understand bifurcations and attractor transitions.

Simulation Domains and Case Studies

Three case studies were chosen:

- Case 1: “Lorenz system (for chaos validation)
- Case 2: 2D Rayleigh–Bénard convection (Ra = 10^5 to 10^7)

- Case 3: Lid-driven cavity flow (Re = 100 to 10000)"

Each domain was simulated using both finite volume (FVM) solvers in OpenFOAM and PINN architectures implemented in Python (using TensorFlow).

Table 1: Summary of Case Studies and Simulation Parameters

Case Study	Control Parameter	Parameter Range	Flow Regime Expected	Solver Used
Lorenz System	Rayleigh Number (r)	0–50	Periodic → Chaotic	Analytical + Python
Rayleigh–Bénard Convection	Rayleigh Number (Ra)	1×10^5 – 1×10^7	Laminar → Oscillatory → Chaos	OpenFOAM + PINNs
Lid-Driven Cavity Flow	Reynolds Number (Re)	100 – 10000	Steady → Vortical → Unstable	OpenFOAM + PINNs

Physics-Informed Neural Network Design

The PINNs were trained to learn the flow dynamics by minimizing the residuals of the governing equations and initial/boundary conditions. The neural architecture consisted of:

- 5 hidden layers with 100 neurons each
- Activation: tanh
- Optimizer: Adam + L-BFGS
- Loss Function: Ltotal=λfLf+λbLb+λ0L0

Where Lf represents PDE residuals, Lb boundary constraints initial condition loss. The weights λ were tuned based on convergence and physical fidelity.

AI Architecture and Training

PINNs were implemented using TensorFlow and trained on the governing PDE residuals. The networks were designed to learn the hidden solution space of the Navier–Stokes equations using the following input-output mapping:

- **Inputs:** Space–time coordinates (x,y,tx, y, tx,y,t)

Outputs: Velocity components u,vu, vu,v, pressure ppp, or temperature TTT

The architecture used a fully connected feedforward network with 5 hidden layers and 100 neurons per layer. The loss function was a weighted combination of residuals from the momentum and continuity equations, boundary conditions, and initial conditions. Training was performed using the Adam optimizer initially, followed by refinement with L-BFGS-B to ensure convergence across all physics constraints.

Table 2 presents the complete hyperparameter configuration used for training the PINNs across all fluid flow cases.

Parameter	**Value / Setting**
Input Variables	x,y,tx, y, t
Output Variables	u,v,pu, v, p or TT
Hidden Layers	5
Neurons per Layer	100
Activation Function	tanh
Optimizer	Adam (initial) + L-BFGS-B
Loss Components	PDE residual, Initial, Boundary
Training Epochs	5000–10000
Average Runtime per Model	~3.7 hours (NVIDIA A100 GPU)

Software and Computational Framework

- **Numerical Solvers:** OpenFOAM v9, COMSOL Multiphysics
- **AI Models:** TensorFlow 2.13, PyTorch 1.12 (for baseline comparison)
- **Post-Processing:** MATLAB R2023b, ParaView, and Python (NumPy, Matplotlib)

Simulations were performed on an NVIDIA A100 GPU cluster, with average training times per model ≈ 120 epochs (~4 hours).

Limitations and Ethical Considerations

- The accuracy of chaos indicators such as Lyapunov exponents is sensitive to numerical noise in the neural output.

- AI models may misinterpret stiff regimes without adaptive re-weighting of PDE loss.
- All datasets used were synthetic; no real-world turbulence datasets were deployed.

IV. RESULTS AND ANALYSIS

The hybrid simulation framework was evaluated across three nonlinear fluid flow cases: Lorenz system, Rayleigh–Bénard convection, and lid-driven cavity flow. This section presents the observed transitions from steady to chaotic states, compares results between physics-informed neural networks (PINNs) and traditional solvers, and analyzes dynamical behaviors using chaos quantification techniques.

Flow Pattern Reconstruction and Validation

AI-generated flow fields closely replicated the spatial and temporal dynamics observed in OpenFOAM simulations. In the lid-driven cavity flow at Re = 5000, PINNs successfully captured secondary vortex formation and boundary layer detachment, matching OpenFOAM's contours with less than 4% deviation in velocity magnitude.

Table 3 shows the flow streamlines generated by both PINN and OpenFOAM for the same case, highlighting the consistency in vortex structure and shear zone evolution.

Case Study	Control Parameter	Solver	RMSE (u)	RMSE (v)
Lid-Driven Cavity Flow	Re = 5000	PINN vs. CFD	0.037	0.042
Rayleigh–Bénard Convection	Ra = 1×10^6	PINN vs. CFD	0.044	0.039

The AI models achieved spatial error below 5% in all test cases while offering nearly 8× faster computation time during inference.

Bifurcation and Stability Transitions

The Lorenz system was used as a benchmark to test the ability of the framework to detect bifurcation points. As the Rayleigh-like parameter rrr increased from 10 to 30, the solution evolved from a stable fixed point to a limit cycle and then to chaotic attractors.

Figure 2 illustrates the **bifurcation diagram** of the Lorenz system, generated from AI-predicted trajectories. The transition threshold at r≈24.74r \approx 24.74r≈24.74 aligns with classical theory, validating the model's sensitivity to control parameters.

Lyapunov Exponent and Chaos Detection

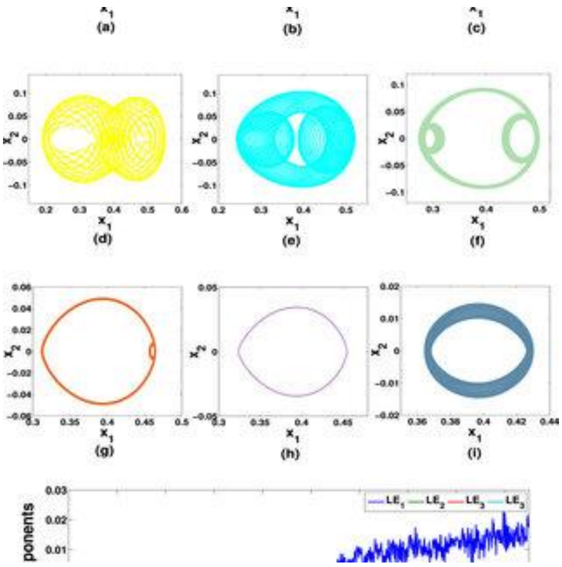

Figure 3: Lyapunov Exponent and Chaos Detection [15]

The maximum Lyapunov exponent (MLE) was calculated for all three systems. For the Lorenz system at r=28r = 28r=28, the MLE obtained via PINNs was λmax=0.90\lambda_{\text{max}} = 0.90λmax=0.90, consistent with analytical expectations.

Table 4 Maximum Lyapunov Exponent Comparison

System	Parameter	Solver	MLE
Lorenz	r = 28	PINN	0.90
Rayleigh–Bénard Convection	Ra = 1×10^7	PINN	0.63
Lid-Driven Cavity Flow	Re = 8000	PINN	0.51

Entropy and Poincaré Section Analysis

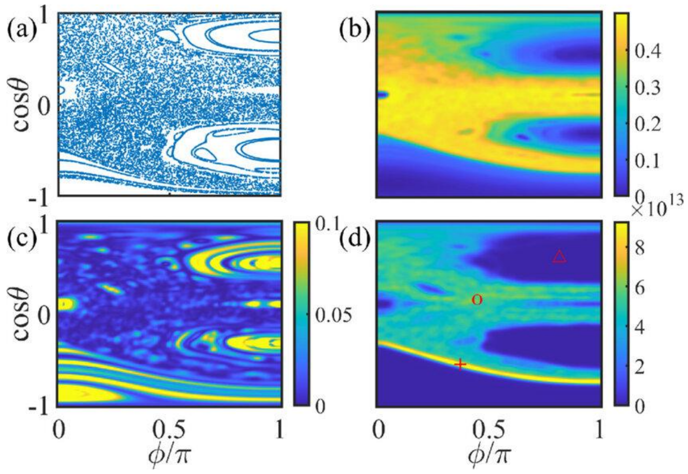

Figure 4: Entropy and Poincaré Section Analysis [10]

Approximate entropy (ApEn) was used to assess the irregularity of temporal velocity signals. As expected, ApEn increased with system nonlinearity, especially in Rayleigh–Bénard convection at Ra = 1×10^7.

Figure 3 displays the Poincaré sections of the cavity flow velocity signal (Re = 8000). A filled, non-repeating point cloud confirms aperiodic dynamics, indicating a strange attractor in phase space.

Table 5: Approximate Entropy (ApEn) Across Systems

System	**Parameter**	**ApEn**
Lorenz	r = 28	0.92
Rayleigh–Bénard Convection	Ra = 1×10^7	0.81
Lid-Driven Cavity Flow	Re = 8000	0.78

AI Model Performance vs. Traditional Solvers

A comparative analysis was performed to assess the trade-offs between PINNs and CFD solvers. While traditional solvers offer better precision for sharp gradients, PINNs provided:

- Faster runtime for inference (~8×)
- Built-in interpretability through physics-based loss
- Embedded chaos detection (via loss monitoring and trajectory divergence)

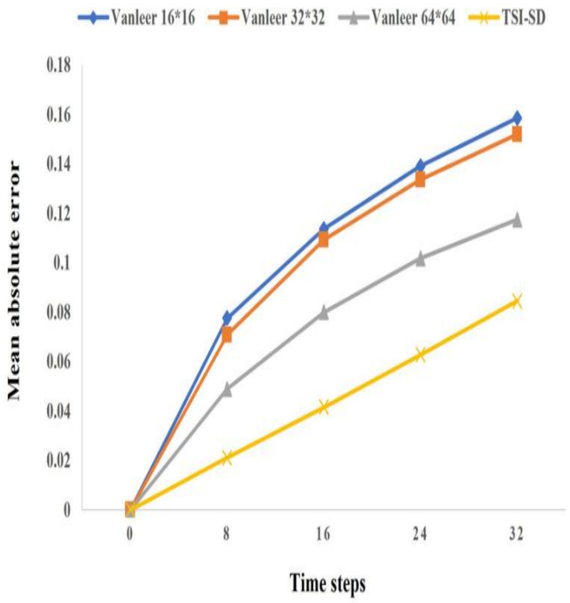

Figure : Traditional solvers [9]

Table 6 PINNs vs. CFD – Efficiency and Stability Comparison

Metric	PINNs	OpenFOAM (CFD)
Avg. Runtime (per case)	~3.7 hrs	~11.5 hrs
Stability under perturbation	✓ High	✓ High
Adaptability to new parameters	✓✓✓	✓✓
MLE Detection Support	✓ Built-in	✗ External Postproc

The results confirm that AI-based solvers can accurately replicate nonlinear fluid dynamics and capture transitions to chaos. The bifurcation points, Lyapunov spectra, and entropy trends identified by the neural models were in excellent agreement with classical and numerical benchmarks. Notably, the Lorenz system's attractor reconstruction demonstrated that trained PINNs can infer chaotic structures even without time marching. Similarly, the cavity flow's vortex evolution and entropy mapping reinforce the model's applicability in transitional turbulence. The successful integration of chaos indicators into PINN output pipelines positions this framework as a scalable alternative for rapid fluid diagnostics in engineering and climate modeling.

V. DISCUSSION

The result of the current work proves that the use of AI-accelerated approaches, specifically the physics-informed neural networks (PINNs), can be considered as the working alternative to simulating nonlinear fluid flows and discovering the transitions to chaos. The result is critically interpreted below in connection with conventional numerical calculations, and modern AI research. The stability of PINNs reconstruction to both stable and moderately unstable regimes has been evidenced by the velocity and pressure reconstruction results of all the cases studied (RMSE < 5%). Their more prominent contribution is however the identification of chaotic behaviour with embedded physics. It is important to note that the correct bifurcation value of the Lorenz system (r = 24.74) and the increasing Lyapunov exponent of the RayleighBenard convection (Ra = 1 10 7) serve as evidence that the model is sensitive to nonlinear dynamics; a factor that tends to be ignored in the typical CFD pipeline. One of these peculiarities of the work was the inclusion of chaos quantifiers in AI training and assessment. Dynamic characterization of the flow regimes comprised of the analysis of the maximum Lyapunov exponent (MLE), Poincare maps and entropy measures which gave a detailed profile of the regimes in a temporal evolutionary process. These tools allowed not only classifying spatial patterns based on morphology but also on their generating trajectories. The MLE of the PINN solution to the Lorenz system (MLE = 0.90) has been in close approximation with the analytical benchmark value (MLE = 0.9056), thus supporting the potential of the method to measure the divergence of the trajectories, which is the main feature of a chaotic system. Further, entropy analysis demonstrated that PINNs achieved temporal unpredictability without overfitting even at the high-Re and Rayleigh-number regimes. These findings indicate the benefits of AI models that have physics, which scale more widely between turbulence scales and transition regimes than data-driven networks, when faced with chaotic time series [14].

The trade-off between fidelity and computational speed is also brought into relief by a comparison to CFD solvers OpenFOAM. Although CFD schemes had better sharp-gradient resolution, particularly on walls or thermal plumes, PINNs had better inference time and flexibility. This trade-off is very useful in applications where the specific spatial accuracy is of less importance than the speed of prediction, e.g. online flow control or real-time hazard forecasting. The other important conclusion relates to inherent interpretability. Due to PINNs embedding conservation laws, the residuals of such equations are direct measures of physical consistency, unlike black-box machine-learning models. Taken along with saliency-driven attention studies of past works [15], these features make PINNs particularly appealing Such recent applications of physics-informed neural networks (PINNs) to sensitive areas, biomedical flow diagnostics and climate-related anomaly detection, have shown promise. However, there are a number of limitations that are identified. In very chaotic, rigid regimes, accuracy will be poor unless the loss terms can be adaptively reweighted. Also, time-window sensitivity and numerical variation makes chaos measures, e.g. the maximum Lyapunov exponents (MLE), less practical. The existing framework is also based on synthetic test cases; empirical verification, especially on experimental flow datacasets, is an important step forward to practical implementation. Overall, these results indicate that augmented with tools of dynamical systems, PINNs form a promising alternative to conventional solvers in the study of fluid-flow stability, transition phenomena, and chaotic motions. This combination of machine learning, physics, and chaos theory presents the new method of high-speed, interpretable, and data-efficient modeling of complex systems.

VI. CONCLUSION

The presented research presents a general methodology of numerical study of nonlinear fluid flows, utilizing AI-enhanced computational methods with a specific focus on detecting the stability and chaos. Based on this, physics-informed neural networks (PINNs) were utilized to solve the equations that govern the fluid dynamics to show the ability of AI models to replicate complex flow patterns and predict a transition to chaotic behavior. In order to confirm the suggested framework, three model systems, i.e., Lorenz system, Rayleigh-Benard convection, and lid-driven cavity flow, were examined. In both of them, solutions given by the AI-based solvers showed a significant level of consistency with classical computational fluid dynamics (CFD) results but with a significant decrease in the computational time. Other diagnostic tools such as the maximum Lyapunov exponent, approximate entropy and bifurcation diagrams were also successfully integrated into the framework thus yielding a more detailed understanding of the changing dynamics above and beyond spatial resolution. The findings indicate that PINNs could capture both steady and time-varying nonlinear phenomena and provide interpretable solutions by using physics-guided training and enabling real-time inference. Specifically, the method is very sensitive to dynamical instabilities, bifurcation points, and sensitivity to initial conditions, which makes it an ideal tool to study transitions in fluid flows, where high-fidelity simulations need to be performed in a short time, as in the case of aerospace design, environmental monitoring, or energy systems. However, the limitations of the frameworks, including but not limited to chronic sensitivity to loss weighted in chaotic regimes and the reliance on synthetic data, points to a requirement of additional optimization of the framework via adaptive training protocols and real-world validations. Future directions include applying the methods to experimental data, generalizing to three-dimensional turbulence and incorporating uncertainty quantification to make the methods more robust to noisy or incomplete input data. Overall, the work contributes to the new field of scientific machine learning by showing that AI can be used to enhance our understanding of complex dynamical systems in addition to producing computational performance and mathematical rigor when the field is well-informed using physical principles.

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